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REACTIVITY COEFFICIENT MEASUREMENT OF BUCKLING

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REACTIVITY COEFFICIENT MEASUREMENT OF BUCKLING

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Introduction: The current series of exponential measurements on light water moderated, slightly enriched uranium rod lattices is intended to provide typical reactor parameters for such assemblies in the clean, cold state. So far, extrapolating to operational conditions in a going power reactor (with fission production buildup, high temperature, structural poisons, partial fuel burnout, Pu buildup, etc.) must be done by calculations which are first made to predict the clean, cold results correctly. Of course, it is desirable to make such estimates of the predicted behavior as sound as possible.

In order to provide some experimental basis for predicting the effect of 25 burnup and 49 buildup, we are planning to supplement our present work with measurements in assemblies fueled partly by Pu. We intend to measure as many quantities as possible, hoping to get critical masses, and to find out pertinent details of the neutron economy.

We have also scheduled some measurements of a similar nature in assemblies fueled by mixtures of 25 and thorium. These should shed light on the utility of such a system as a 25-23 converter.

Since fabrication of such special rods is expensive, we have been trying to find ways to measure the quantities we are interested in, using only small amounts of fuel. This report gives the results of a measurement of buckling done by finding the reactivity coefficient of an unknown fuel in terms of a known fuel.

Method: Basically the measurement consists in replacing the center of a reproducing assembly by a small reproducing region having an unknown buckling. The critical mass is measured before and after the substitution, and the change in critical mass is used as a measure of the unknown buckling. The precise value of the buckling could, in principle, be found in two ways. The measurement could be calibrated by using in place of the unknown a series of fuels of different but known B^2; the unknown value could then be found from a curve of B^2 versus change in critical mass. If the unknown center region is large enough, it may, on the other hand, be possible to determine the buckling from a two-region calculation.

In the test measurement we have made, the known region contained .6" diameter uranium rods enriched to 1.15% 25, and the "unknown" fuel consisted of 19 .6" diameter rods of uranium enriched to 1.027%. The moderater was light water. An effectively infinite water reflector was used. The bucklings of both enrichments at the water-to-metal volume ratio used (3:1) were known beforehand, and the purpose of the measurement was to find out if simple two-region analysis would be adequate for predicting the B^2 of the 1.027% enrichment rod region.

The critical masses for the two cases were found by a method described in BNL Log No. C-7592. The uranium was loaded, with suitable precautions, in the presence of a neutron source until the multiplication of neutrons from spontaneous fissions was great enough to permit flux level measurements.

At this point the neutron source was removed and the neutron flux in the assembly was measured as rods were removed from the periphery. The result of the measurement was then an experimental plot of the local neutron flux as the function of the loading.

Because of spontaneous fissions in a simple one-region assembly the flux density in the core has the form (we include the effect of the reflector through an assumed constant reflector savings):

$$\phi = \sum_{j,k=1}^{\infty} C_{j,k} \sin \frac{\pi j z}{h} J_o\left(\frac{\xi_k r}{R+\lambda}\right) \tag{1}$$

with

$$C_{j,k} = \frac{12 \left\{1 - (-1)^j\right\} Q}{\pi j \lambda_t \xi_k J_1(\xi)_k \left\{\left(\frac{\xi_k}{R+\lambda}\right)^2 + \left(\frac{k\pi}{h}\right)^2 - B^2\right\}} \tag{2}$$

Q = source of spontaneous fission neutrons
h = height of the assembly
ξk = successive zeroes of $J_o(u)$
λt = transport mfp
λ = reflector savings

For the nearly critical lattices we have used, only the fundamental mode j,k = 1 is important,[1] so that

$$\phi = \frac{\text{constant} \cdot \sin \frac{\pi z}{h} J_o\left(\frac{\xi_1 r}{R+\lambda}\right)}{\left(\frac{\xi_1{}^2}{R+\lambda}\right)^2 + \left(\frac{\pi}{h}\right)^2 - B^2} \tag{3}$$

but

$$B^2 = \left(\frac{\xi_1}{R_c+\lambda}\right)^2 + \left(\frac{\pi}{h}\right)^2 \tag{4}$$

(R_c the critical loaded radius) for a critical finite cylinder. Thus

$$\phi = \frac{\text{constant} \cdot \sin \frac{\pi z}{h} J_o\left(\frac{\xi_1 r}{R+\lambda}\right)}{\left(\frac{\xi_1}{R+\lambda}\right)^2 - \left(\frac{\xi_1}{R_c+\lambda}\right)^2} \tag{5}$$

The thermal flux is measured at a fixed point on the central axis. Thus a plot of $1/\phi$ vs. $\left(\frac{1}{R_c+\lambda}\right)^2$ is a straight line intersecting the axis at

$$\frac{1}{(R+\lambda)^2} = \frac{1}{(R_c+\lambda)^2} \tag{6}$$

1. The fluxes measured had .01% harmonic content.

Of course, the procedure is essentially that of the critical assembly. But since the lattices are not loaded all the way, the analytical justification for extrapolating the flux plots to critical must be made.

In the two region assembly the experimental procedure was precisely the same. Analysis of the experimental results, however, should strictly be done by a two-region calculation. We did not do this. What we did was to establish that the experimental plot of $\frac{1}{\phi}$ vs $\frac{1}{(R+\lambda)^2}$ is in this case a straight line also, and assume that the intersection of this straight line with the axis correctly indicates the critical mass.

Experimental procedure: The lattices were loaded up to a suitable k_{eff}, with appropriate flux monitors, safety rod, and trip circuits, and with flux levels maintained by multiplication of neutrons from a Po-Be source. At this point the source was removed from the lattice, and taken away from the vicinity. A small BF_3 counter was inserted in the core, and count rates were recorded for several smaller loaded radii.

The maximum count rates from spontaneous fission multiplication were about 250 c/m and a range of about a factor of 3 in neutron multiplication was used.

Analysis: As has been mentioned earlier, all loadings were done with an effectively infinite water reflector. The simplest calculation of the critical mass of the assembly with the center replaced with an "unknown" is done with ordinary one-group, two-region diffusion theory, with the effect of the reflector being lumped into an assumed constant reflector-savings.

In this case, if we let the subscripts 1 and 2 represent the inner and outer region, respectively, the pile equations are

$$\nabla^2 \phi_1 + B_1^2 \phi_1 = 0$$
$$\nabla^2 \phi_2 + B_2^2 \phi_2 = 0 \qquad (7)$$

The solutions are

$$\phi_1 = \left[a \ J_o (ur) + b \ Y_o (ur) \right] \cos \frac{\pi z}{h} \tag{8}$$

$$\phi_2 = \left[c \ J_o (vr) + d \ Y_o (vr) \right] \cos \frac{\pi z}{h}$$

with

$$B_1^2 = u^2 + \left(\frac{\pi}{h}\right)^2$$
$$B_2^2 = v^2 + \left(\frac{\pi}{h}\right)^2 \tag{9}$$

The boundary conditions are

$$\phi_1 = \text{finite at } r = 0$$
$$\phi_1 = \phi_2 \text{ at } r = R_1 \text{ (interface flux)} \tag{10}$$
$$\frac{d\phi_1}{dr} = \frac{d\phi_2}{dr} \text{ at } r = R_1 \text{ (interface current)}$$
$$\phi_2 = 0 \text{ at } r = R_2 \text{ (extrapolated boundary)}$$

In the third boundary condition we have assumed the equality of the diffusion coefficients for the two regions. In general, of course, we may not assume this, but the two-region assembly we are analyzing here very closely satisfies this condition.

Insertion of the solutions (8) into the boundary conditions (10) yields readily the secular equation

$$\frac{u \ J_1 \ (u \ R_1)}{J_o \ (u \ R_1)} = \frac{v \left[J_1 (v R_1) Y_o (v R_2) - Y_1 (v R_1) J_o (v R_2) \right]}{Y_o (v R_2) J_o (v R_1) - Y_o (v R_1) J_o (v R_2)} \tag{11}$$

Supposing that R_1 and R_2 are known and that v is known from (9), we may solve for u. Then use of (9) gives B_1^2, the unknown buckling.

We have also carried through the two-group, two-region analysis. For the experimental case we are analyzing, the two-region diffusion coefficients and diffusion lengths are very nearly the same in the outer and inner region. If we assume they are exactly equal, then the two-group secular equation reduces again to (10).

The experiment and its analysis are, stepwise, as follows:

(1) A value of λ is assumed from previous exponential experiments.

(2) The neutron flux resulting from multiplication of the spontaneous fission neutrons is measured for the one-region assembly as a function of loaded radius R.

(3) A straight line plot of $1/\phi$ vs $\left(\frac{1}{R+\lambda}\right)^2$ is made, and the value of the critical radius R_c is found from the intercept of this straight line.

(4) B_2^2 is evaluated as

$$B_2^2 = \left(\frac{\xi_1}{Rc}\right)^2 + \left(\frac{\pi}{h}\right)^2 \tag{12}$$

(5) The inner region of this assembly with radius R_1, is then replaced by the unknown, and steps (2) and (3) are repeated. The critical radius in this case is R_2.

(6) These values of B_2^2, R_1, R_2 are then used in equations (9) and (11) to give B_1^2.

<u>Results</u>: The value of λ used was 6.68 cm. This value was found from previous exponential experiments with the 3:1 volume ratio lattice of 1.15% enriched rods in ordinary water. The experimental inverse flux plots of the spontaneous neutron multiplication are shown in figure 1. It is evident that the points in both cases are well represented by straight lines. This is, of course, to be expected for the one-region assembly, because equation (5) predicts this behavior.

A straight-line fit to the two-region data seems equally justified by the appearance of the plotted points.

The critical radii for the two cases are found from the intercepts of least-squares fit straight lines. They are

$$R_c = 37.498 \pm .007 \text{ cm (one-region)}$$

$$R_2 = 38.088 \pm .064 \text{ cm (two-region)}$$

with the error limits established by the standard deviations from the best straight lines. Actually, the accuracy of the measurements is probably closer to the latter value; the points for the one-region measurement fit the straight line better than would be expected just from the ± 1% counting accuracy.

We find

$$v = \frac{2.4048}{R_c} = 6.4131 \times 10^{-2} \text{ cm}^{-1}$$

The chosen value of R_1 is based on the volume ratio (3:1) and the number of rods used in the central region (19). We get

$$R_1 = 6.8325 \text{ cm}$$

The value of u is obtained from inserting these values for R_1, R_2, and v in (9) and (11). It is

$$u = 5.2755 \times 10^{-2} \text{ cm}^{-1}$$

The height h of the assembly is taken from geometry to be 133.6 cm. Thus we get

$$B_1^2 = (5.2755 \times 10^{-2})^2 + \left(\frac{\pi}{133.6}\right)^2$$

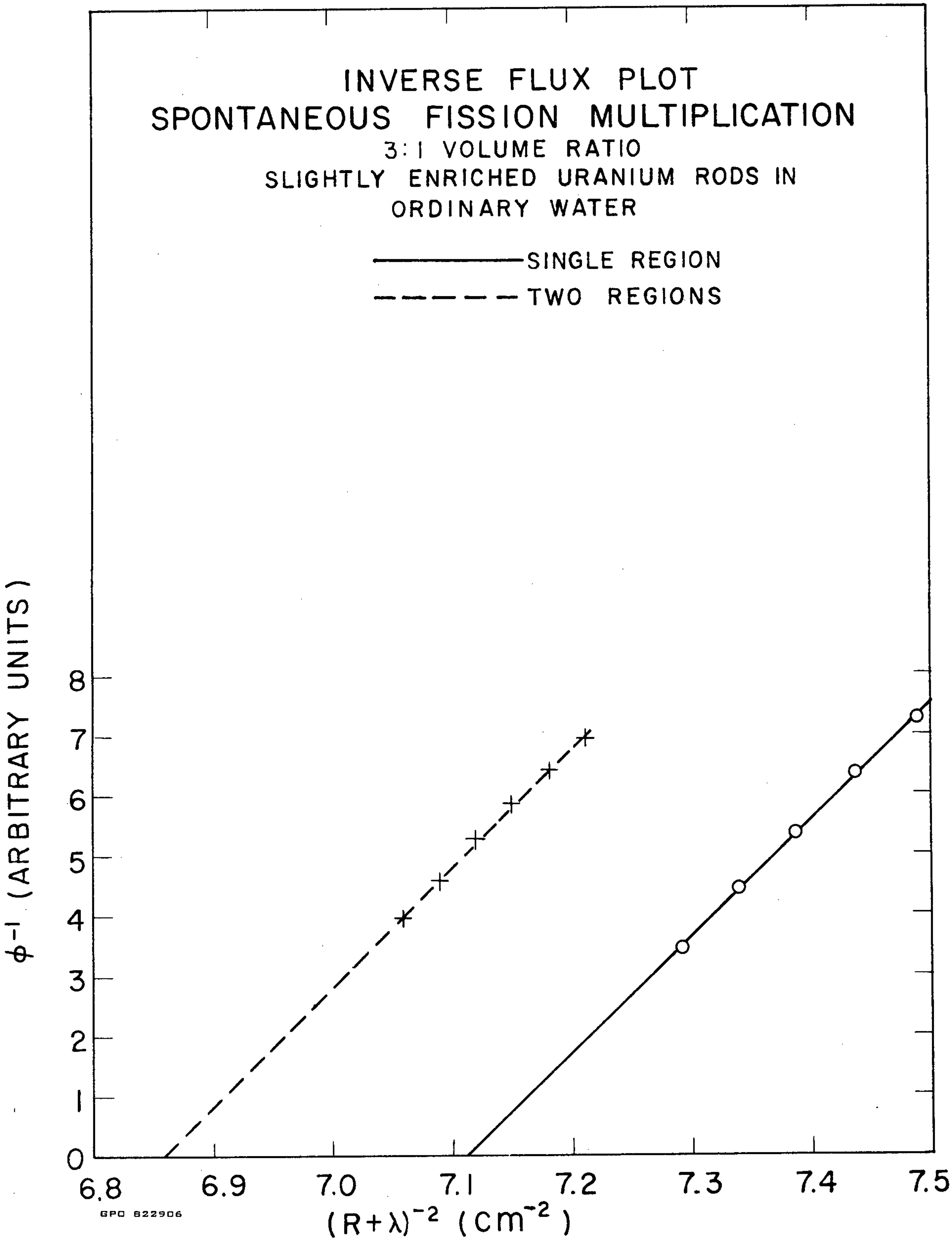
$$= 33.36 \pm .50 \times 10^{-4} \text{ cm}^{-2}$$

with the error estimate based eventually on the least squares fits to the inverse flux plots. This value is to be compared with that determined from previous exponential experiments:

$$B_1^2 = 32.93 \pm .34 \times 10^{-4}\ cm^{-2}$$

The two values agree to 1.3%, and are within the mutual error limits.

Certainly a large part of this excellent agreement can be associated with the similar neutron slowing-down and diffusion characteristics of the two regions. If they were not similar, the two-group analysis would not give the same value of B^2 as the one group analysis, and results would be more uncertain. We may expect then that our scheduled measurements with alloyed 1% Pu, 99% U rods will produce good values of B^2. Those made with the thorium-uranium rods may not be as trustworthy. We expect to find out something about their validity by measuring with two enrichments of uranium in the outer region.


INVERSE FLUX PLOT
SPONTANEOUS FISSION MULTIPLICATION
3:1 VOLUME RATIO
SLIGHTLY ENRICHED URANIUM RODS IN
ORDINARY WATER
SINGLE REGION
TWO REGIONS
ϕ^{-1} (ARBITRARY UNITS)
0
1
2
3
4
5
6
7
8
6.8
6.9
7.0
7.1
7.2
7.3
7.4
7.5
$(R+\lambda)^{-2}$ (cm^{-2})

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